647510 **BSc4MatC0401** **Seat No: ___________**

B.Sc. Semester - 4 (CBCS) Examination

March/April- 2019

MATHEMATICS

(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1

(A) ATTEMPT ANY ONE **07**

(1) Prove that a non-empty subset W of a vector space V is a subspace
$$iff \alpha w_1 + \beta w_2 \in W, \forall w_1, w_2 \in W \text{ and } \forall \alpha, \beta \in R$$

(2) Prove that a set of vectors $S = \{v, v_1, v_2, \ldots, v_n\}$of a vector space V is linearly dependent $iff \exists$ a vector $v_k \in S, 2 \le k \le n \ni v_k$ is a linear combination of its preceding vectors $v_1, v_2, \ldots, v_{k-1}$.

(B) ATTEMPT ANY ONE **04**

(1) By means of an example show that the union of two subspaces of a vector space may not be a subspace.

(2) Show that the set $V$of all real valued continuous functions defined on $[0,1]$ is a vector space over R with respect to the binary operations of vector addition and scalar multiplication defined as follows:
$$(f+g)(x) = f(x) + g(x); \forall f, g \in V \wedge \forall x \in [0,1]$$
$$(\alpha f)(x) = \alpha f(x); \forall f \in V, \forall \alpha \in R \text{ and } \forall x \in [0,1] \text{ .}$$

(C) ATTEMPT ANY ONE **03**

(1) Prove that a finite superset of a linearly dependent set of vectors of a vector space is also a linearly dependent set.

(2) Define: Spanning set
Find the spanning set of $W = \{(a,b,c) \in R^3 / a + b + c = 0\}$ of the vector space R^3.

Que-2

(A) ATTEMPT ANY ONE **07**

(1) Prove that any linearly independent set of vectors of a vector space can be extended to form a basis of the vector space.

(2) If W_1 and W_2 be any subspaces of a finite dimensional vector space V, then prove that $Dim(W_1 + W_2) = Dim(W_1) + Dim(W_2) - Dim(W_1 \cap W_2)$.

(B) ATTEMPT ANY ONE **04**

(1) Define: Basis of a vector space.
Prove that there are equal number of vectors in any two basis of a vector space.

(2) Show that $W_1 + W_2 = R^4$ by finding $Dim(W_1), Dim(W_2)$ and $Dim(W_1 \cap W_2)$ where
$W_1 = \{(a,b,c,d) \in R^4 / a + c - d = 0\}$ and
$W_2 = \{(a,b,c,d) \in R^4 / a + 2b = 0\}$ are subspaces of the vector space R^4.

(C) ATTEMPT ANY ONE **03**

(1) If W is a subspace of a vector space V, then prove that $Dim(W) \le Dim(V)$.

(2) Extend $A = \{(1,0,0), (-1,2,3)\}$ upto a basis of the vector space R^3.

Que-3

(A) ATTEMPT ANY ONE **07**

(1) State and prove the rank-nullity theorem for linear transformations.

(2) Prove that $L(U,V) = \{T: U \to V / T \text{ is a Linear Transformation}\}$ forms a vector space over R with respect to addition and scalar multiplication of linear transformations.

(B) ATTEMPT ANY ONE **04**

(1) A linear transformation $T: U \to V$ is one-one $iff\ N_T = \{\theta\}$.

(2) If $T: V \to V$ be a linear transformation such that $T^2 + T + I = O$, then prove that T is non-singular.

(C) **ATTEMPT ANY ONE** **03**

(1) Show that $T: R^3 \to R^4, T(a,b,c) = (a+b, b+c, c+a, a+b+c), \forall (a,b,c) \in R^3$ is a linear transformation.

(2) Show that the linear transformation $T: R^2 \to R^2, T(a,b) = (acos\theta - bsin\theta, asin\theta + bcos\theta), \forall (a,b) \in R^2$ is non-singular and find T^{-1}.

Que-4

(A) **ATTEMPT ANY ONE** **07**

(1) Let $T: V \to V$ be a linear transformation and B be any basis of V. Then, prove that T is singular $det([T;B]) = 0$.

(2) Define: Eigen values & Eigen vectors of a linear transformation. Find eigen values and eigen vectors of the linear transformation $T: R^3 \to R^3$ defined as $T(x,y,z) = (x+y+z, x+y+z, x+y+z), \forall (x,y,Z) \in R^3$.

(B) **ATTEMPT ANY ONE** **04**

(1) Let $T: V \to V$ be a linear transformation and let $Dim(V) = n$. Then, if $\lambda_1, \lambda_2, \dots, \lambda_n$ are distinct eigen values of T and if $B = \{v_1, v_2, \dots, v_n\}$ be the corresponding T-eigen basis of V, then prove that $[T;B] = diag(\lambda_1, \lambda_2, \dots, \lambda_n)$, where v_i is the eigen vector of T corresponding to the eigen value $\lambda_i, \forall i, 1 \le i \le n$.

(2) How many eigen vectors may exist corresponding to an eigen value of a linear transformation? **Justify** .

(C) **ATTEMPT ANY ONE** **03**

(1) Define: (i) Linear Functional
(ii) Dual of a vector space
(iii)Adjoint of a linear transformation

(2) Let $T: R^3 \to R^4, T(a,b,c) = (a+b, b+c, c+a, a+b+c), \forall (a,b,c) \in R^3$ be a linear transformation.
Find $[T; B_1, B_2]$ where B_1 and B_2 are standard basis of R^3 and R^4 Respectively.

Que-5

(A) **ATTEMPT ANY ONE** **07**

(1) Prove that the radius of curvature of a polar curve $r = f(\theta)$ is given by $\rho = \frac{(r^2+{r_1}^2)^{\frac{3}{2}}}{r^2 - rr_1 + 2{r_1}^2}, where r_1 = \frac{dr}{d\theta}$ and $r_2 = \frac{d^2r}{d\theta^2}$.

(2) Find the oblique asymptotes of the general algebraic curve.

(B) **ATTEMPT ANY ONE** **04**

(1) Find the radius of curvature at any point (x,y) on the curve $y^2 = 4ax$.

(2) Find all asymptotes of the curve $xy^2 + y^3 = x + y + 1$.

(C) **ATTEMPT ANY ONE** **03**

(1) Define: (i) Node
(ii) Cusp
(iii) Isolated point

(2) Find multiple points on the curve $x^3 + y^3 - 3x^2 - 3xy + 3x + 3y - 1 = 0$ and determine the types of it.

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